Formalization of Rice's Theorem for a Turing Complete Functional Language Model

Thiago Mendonça Ferreira Ramos (thiagomendoncaferreiraramos@yahoo.com.br)
Universidade de Brasília, Instituto de Ciências Exatas, Departamento de Ciência da Computação

Abstract. Classical proofs of Rice's Theorem assume the existence of a universal (Turing) machine and build a reduction from the problem of deciding whether a machine halts or not to the problem of separability of semantic properties of machines. This work presents a formalization in PVS of Rice's Theorem for a computational model given as a class of partial recursive functions. The model is build over basic operators that when restricted to successor, projections, greater than and bijection from tuples of naturals to naturals, results in a model that is formalized to be Turing complete. The main differences with classical proofs are that the given formalization is developed for a functional programming model and that the proof does not depend on the undecidability of the Halting Problem, being made directly without using any translation to or from other computational models. As corollaries, straightforward formalizations of the undecidability of the Halting Problem, functional equivalence problem, existence of fixed points problem and self-replication problem are obtained.

References

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